Influence of Far-Field Boundary on the Calculated Flowfield and Performance of Rotors

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Abstract

A COMPUTATIONAL experiment was conducted to demonstrate the influence of the physical location of the far-field boundary on the calculated performance and flowfield of a two-dimensional Darrieus rotor. A substantial sensitivity to the extent of the computational domain was noted and a series of calculations and comparisons with other approaches are presented to show the influence on pressure recovery and power estimates. Although the study was restricted to the case of a rotor, the sensitivity to far-field boundary will be of concern to the computation of other momentum devices.

Introduction

A two-dimensional Darrieus wind turbine, facing a uniform wind and spinning in the anticlockwise direction, is assumed to be placed at the center of a Cartesian domain. The computational domain is subdivided into control volumes by a series of orthogonal grid lines. The rotor-blades paths are circular in any horizontal plane. The velocity and presure field were obtained by solving steady, laminar, incompressible, Navier-Stokes equations using a primative variable finite-difference procedure known as SIMPLER.[1]

The influence of the spinning turbine blades was introduced as momentum source terms S_x and S_y (not known a priori) in the conservation equations. These terms are zero everywhere except at the circle described by the path of the blades. Rajagopalan and Fanucci[2] proved this approach to be feasible in an application to inviscid flow around a two-dimensional vertical axis wind turbine in cylindrical coordinates, and further validation was obtained by Paraschivoiu et al.[3] who compared the predictions with experiments after including dynamic stall effects.

In functional notation, for a time-averaged calculation, the source terms can be written as

$$S_x = S_x(C_l, C_d, \alpha, V_{\text{abs}}, \omega, R, \theta, c, \rho, B) \tag{1a}$$

$$S_y = S_y(C_l, C_d, \alpha, V_{\text{abs}}, \omega, R, \theta, c, \rho, B) \tag{1b}$$

where C_l and C_d are airfoil characteristics of the turbine blade, α is the angle of attack of the blade to the relative velocity vector, V_{abs} is the absolute velocity of the fluid at the location (R, θ), ω is the angular velocity of the rotor, c is the chord of the blade, and B is the number of blades. The dependence of S_x and S_y on Reynolds number is considered only implicitly through the airfoil sectional characteristics C_l and C_d. The source terms, evaluated using the procedure illustrated in Ref. 2, resolved for Cartesian directions are

$$S_x = \left(\frac{Bc\rho\Delta\theta V_{\text{rel}}}{4\pi}\right)[(C_d v_r' - C_l v_\theta')\cos\theta - (C_d v_\theta' + C_l v_r')\sin\theta] \tag{2a}$$

$$S_y = \left(\frac{Bc\rho\Delta\theta / V_{\text{rel}}}{4\pi}\right)[(C_d v_r' - C_l v_\theta')\sin\theta - (C_d v_\theta' + C_l v_r')\cos\theta] \tag{2b}$$

where v_r' and v_θ' are the components of V_{rel} the relative velocity seen by the blade-fixed (r, θ) cylindrical system and $\Delta\theta$ is the arc length included in a cell.

After a grid-independence study, a 24 × 24 grid was used in the region near the turbine and the overall grid for the entire domain, including the central fine grid, was 78 × 78. The total number of grid points was kept constant in order to keep round-off and machine errors constant. The nondimensional blade tangential force $FT+$ defined as

$$FT+ = (C_l \sin\alpha - C_d \cos\alpha)(V_{\text{rel}}/V_\infty)^2$$

is compared with the predictions of the double multiple streamtube model CARDAVV (Ref. 4) in Figs. 1 and 2. In the preceding equation V_∞ is the freestream wind velocity. The overall domain was studied at two sizes, a smaller one whose extent was 12 rotor diameters in both x and y directions giving a blockage area of about 8%, and a larger one of 96 diameters with a blockage of about 1%. The larger computational domain was generated by stretching the grid in the region outside of the central region. The fine mesh at the turbine was not changed in order to not vary the accuracy of the force computations. The velocity at the y boundaries was taken tangent to the boundary and equal to the freestream value. The velocity over the inlet boundary is normal to the boundary and equal to the freestream value.

In general, there is good agreement between the two models for the larger domain except at the upwind side, near-azimuthal angle 180 deg, where the force is a maximum. For the smaller domain, however, there is a marked improvement in the agreement near the 180-deg position. It is expected that the solution using the larger computational domain will have a smaller boundary influence on the force field computation and therefore represents a more accurate solution. Momentum models, in general, do not account for streamtube divergence through the rotor and assume the wake pressure to be the same as freestream pressure. As a consequence, momentum-based methods such as those of Ref. 4 tend to overpredict performance for certain cases such as heavily loaded rotors. The use of a small computational domain also restricts streamtube divergence giving the apparent agreement between the two methods.

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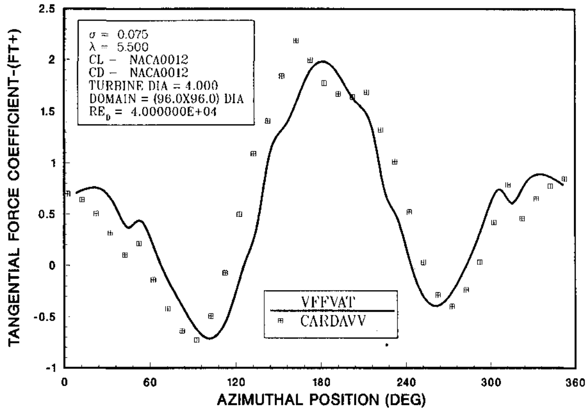

Fig. 1 Variation of $FT+$ in the larger domain.

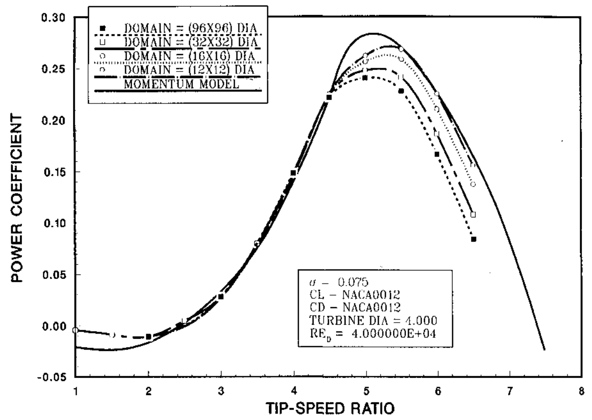

Fig. 3 Performance prediction for different domain sizes.

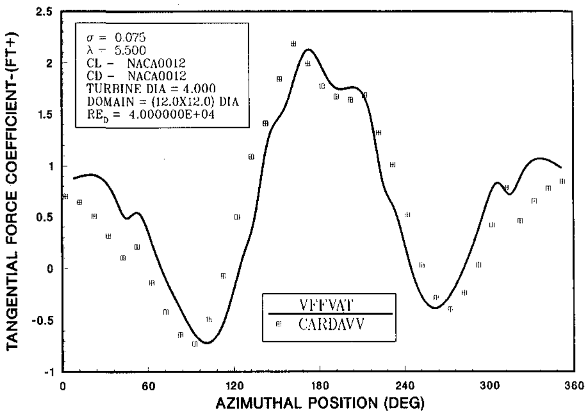

Fig. 2 Variation of $FT+$ in the smaller domain.

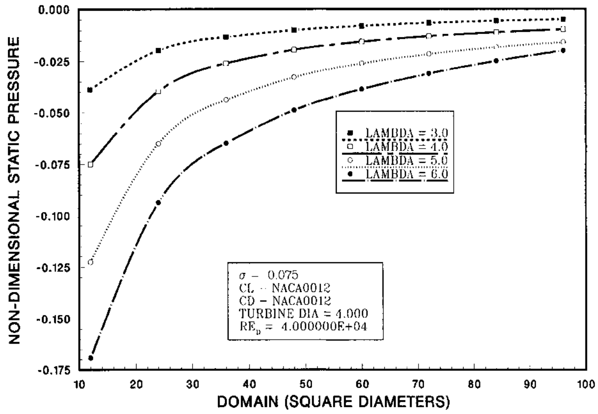

Fig. 4 Static pressure recovery at the center of the exit plane.

The estimation of tangential force coefficient directly affects the power-performance calculation for the turbine. The power coefficient of the turbine C_p (ratio of the power extracted by the turbine to the power in the incoming flow) was calculated for four different domain sizes, with grids stretched in the same way as for the earlier comparison. The results are shown in Fig. 3, where it is seen once again that the smallest domain solution agrees better with the momentum theory predictions of Ref. 5.

A calculation of the static pressure variation across the exit plane shows a pressure recovery less than the freestream for the smallest domain, but it improves as the domain size is increased. Thus, the pressure field imposed by the outer boundary has considerable effect on the performance as well as the flowfield. It was found that the effect of the blade tip-speed ratio λ (the ratio of the linear velocity of the blades to the freestream velocity) combined with the domain size appears to play an important role in the exit pressure recovery. The exit pressure coefficient results for different tip-speed ratios and domain sizes are presented in Fig. 4. The exit static pressure recovery is found to be the least for the case of the smallest domain and the largest tip-speed ratio.

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